Lecture Notes
On
Design of Machine Elements
5th Semester
Session 2024-2025 (W)

By
Mr. Baneswar Munda
(Sr. Lecturer)
Department of Mechanical Engineering
Government Polytechnic, Sonepur

CHAPTER 1

INTRODUCTION:

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived.

The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

Classifications of Machine Design:

The machine design may be classified as follows:

1. Adaptive design: In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training.

The designer only makes minor alternation or modification in the existing designs of the product.

2. Development design: This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. New design: This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design.

The designs, depending upon the methods used, may be classified as follows:

(a) Rational design: This type of design depends upon mathematical formulae of principle of mechanics.

(b) Empirical design: This type of design depends upon empirical formulae based on the practice and past experience.

(c) Industrial design: This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) Optimum design: It is the best design for the given objective function under the specified constraints. It may be achieved by minimising the undesirable effects.

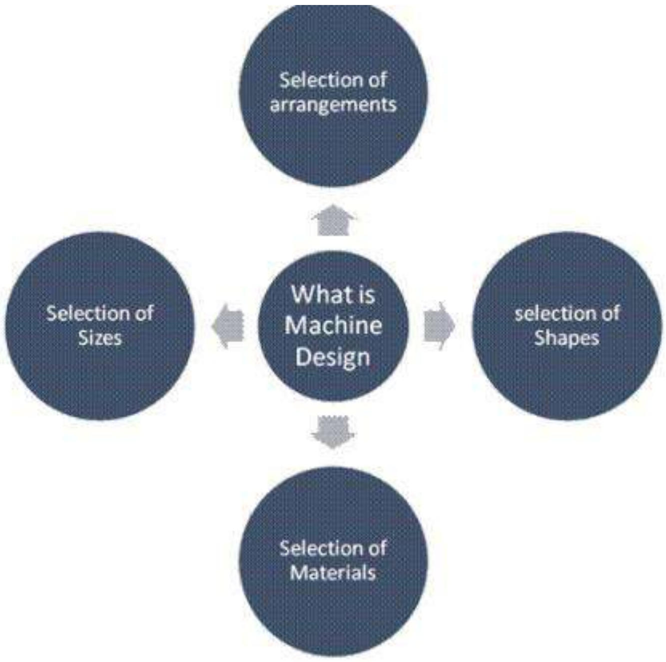
(e) System design: It is the design of any complex mechanical system like a motor car.

(f) Element design: It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(g) Computer aided design: This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimisation of a design.

Factors governing the design of machine elements:

Factors to be considered for selection of material for design of machine elements
a) Availability: Material should be available easily in the market.
b) Cost: the material should be available at cheaper rate.
c) Manufacturing Consideration: the manufacturing play a vital role in selection of material and the material
should suitable for required manufacturing process.
d) Physical properties: like color, density etc.
e) Mechanical properties: such as strength, ductility, Malleability etc.
f) Corrosion resistance: it should be corrosion resistant.



General Procedure in Machine Design:

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways.

However, the general procedure to solve a design problem is as follows:

Fig. General Machine Design Procedure

1. Recognition of need: First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. Synthesis (Mechanisms): Select the will give the desired motion.

3. Analysis of forces: Find the forces acting on each member of the machine and the energy transmitted by each member.

4. Material selection: Select the material best suited for each member of the machine.

5. Design of elements (Size and Stresses): considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. Modification: Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. Detailed drawing: Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. Production. The component, as per the drawing, is manufactured in the workshop. The flow chart for the general procedure in machine design is shown in Fig.

Engineering Materials:

The engineering materials are mainly classified as:

1.Metals and their alloys, such as iron, steel, copper, aluminum, etc.
2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:
1**. Ferrous metals**.

The Ferrous metals are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

2**. Non-ferrous metals.**

The Non-ferrous metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost.

The following factors should be considered while selecting the material:
• Availability of the materials.
• Suitability of the materials for the working conditions in service.
• The cost of the materials.
Mechanical Properties of Metals:

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include; Strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness.

1. Strength: It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

2. Stiffness: It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3. Elasticity: It is the property of a material to regain its original shape after deformation when the external forces are removed.

4. Plasticity: It is property of a material which retains the deformation produced under load permanently.

5. Ductility: It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice are mild steel, copper, aluminum, nickel, zinc, tin and lead.

6. Brittleness: It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Cast iron is a brittle material.

7. Malleability: It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. The malleable materials commonly used in engineering practice are lead, soft steel, wrought iron, copper and aluminum.

8. Toughness: It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated.

9. Resilience: It is the property of a material to absorb energy and to resist shock and impact loads. This property is essential for spring materials.

10. Creep: When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep. This property is considered in designing internal combustion engines, boilers and turbines.

11. Fatigue: When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. This property is considered in designing shafts, connecting rods, springs, gears, etc.

12. Hardness: It is the property of the metals; it adopts many different properties such as resistance to wear, scratching, deformation and machinability etc. The hardness of a metal may be determined by the following tests:

a) Brinell hardness test.
b) Rockwell hardness test.
c) Vickers hardness test.

Working Stress:
When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the working stress.

Factor of Safety:
It is defined, in general, as the ratio of the maximum stress to the working stress. Mathematically, Factor of safety = Maximum stress / Working or design stress

• In case of ductile materials; e.g. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress.

In such cases;

Factor of safety = Yield point stress / Working or design stress

• In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials.

Therefore, the factor of safety for brittle materials is based on ultimate stress.

In such cases:
Factor of safety = Ultimate stress / Working or design stress

Stress Strain Curve for Mild Steel:

When a ductile material like mild steel is subjected to tensile force, it undergoes different stages before failure. Stress strain curve is the graphical representation of this stages. Different material may have different curve. Usually ductile materials follow similar pattern. so is for brittle materials. Here is the explanation of stress strain curve for mild steel which is ductile material.

Here is the list of different stages when ductile material subjected to force till its failure.
• Proportional limit (point A)
• Elastic limit (point B)
• Yield point (upper yield point C and lower yield point D)
• Ultimate stress point (point E)
• Breaking point (point F)

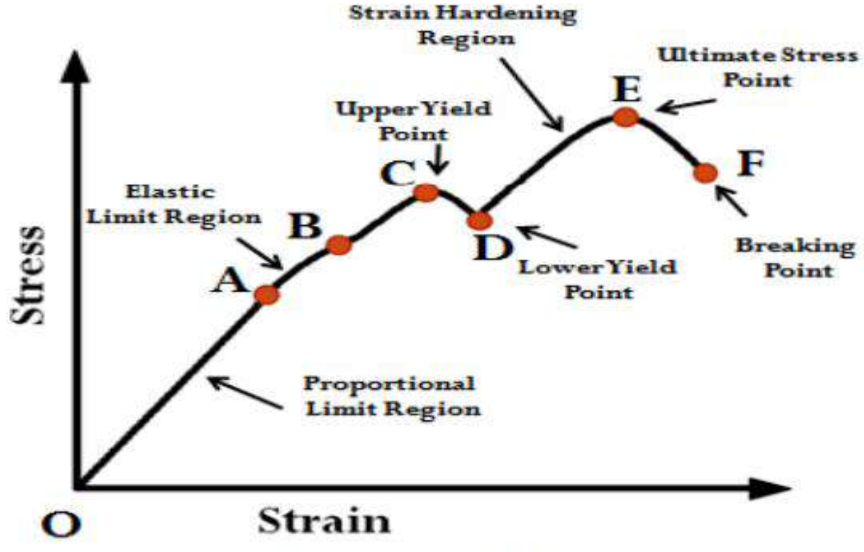


Fig.Stress-strain curve for mild steel

Proportional limit:
As shown in stress strain curve for mild steel, up to the point A, stress and strain follow a relationship. This is known as Hook's law. Up to the limit of proportionality, stress directly followed the strain. This means ratio of stress and strain remains constant

Elastic limit:
Up to this limit (point B), is material will regain its original shape is unloaded. Point B is known as elastic point.

Yield limit:
When material is loaded beyond its elastic limit, it will not regain its original shape. There will be always some deformation.

Ultimate stress:
This is the maximum stress a material can bear. Value of stress corresponds to peak point on stress strain curve for mild steel is the ultimate stress. It is denoted by point E in diagram.

Breaking stress:
Point on the stress strain curve where material fails, is known as breaking point. Stress correspond to this point is known as breaking stress.

<u>Modes of failure:</u>

A mechanical component may fail i.e. it may be unable to perform its function satisfactorily, as a result of any one of the following three modes of failure:

1. Failure by elastic deflection.
2. Failure by Yielding
3. failure by Fracture

1. Failure by elastic deflection

• In applications like transmission shaft supporting the gears, the maximum force acting on the shaft, without effecting it performance is limited by the permissible elastic deflection. Sometimes the elastic deflection results in unstable conditions, such as buckling of columns or vibrations. The design of mechanical component, in all these cases, is based on the permissible lateral or torisional deflection. The stresses induced in the component are not significant and properties of the material are not of primary importance. The moduli of elasticity and rigidity are the important properties and dimension of component are determined by the load deflection equation.

• In short, in a components like : columns, beams, shafts etc.,the torsional deflection in an elastic region is termed as failure of the component

2. Failure by Yielding

• For ductile material deformation occurs after the yield point, resulting in permanent deformation of the machine element which ultimately breaks at breaking point. Hence for ductile materials, failure is usually considered to have occurred when yielding i.e. plastic deformation reach a limit, when engineering usefulness of the part is destroyed, even through there is no rupture or fracture of machine part. Thus, the yield point is criterion of failure of ductile materials subjected to static loading.

• In short, when a mechanical component, made of ductile material, undergoes yielding or plastic deformation, its functional utility comes to an end and it is termed as failure of the component. Such failure is known as elastic failure.

3. Failure by fracture

• In case of brittle materials the yield point and ultimate strain is very nearly equal to unity. So brittle materials are considered to have failed by fracture with little or no permanent deformation.

• Sudden separation or a breakage of a material along the cross-section normal to the direction of stress is known as fracture. Fracture is a sudden failure without plastic deformation. The failure of components made of brittle material is due to fracture

CHAPTER 2

Design of Fastening Elements

Welded Joint:

A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding.

Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted joints. It is also used as a repair medium e.g. to reunite metal at a crack, to build up a small part that has broken off such as gear tooth or to repair a worn surface such as a bearing surface.

Advantages and Disadvantages of Welded Joints over Riveted Joints:

1.The welded structures are usually lighter than riveted structures. This is due to the reason, that in welding, gussets or other connecting components are not used.
2. The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.
3. Alterations and additions can be easily made in the existing structures.
4. As the welded structure is smooth in appearance, therefore it looks pleasing.
5. In welded connections, the tension members are not weakened as in the case of riveted joints.
6. A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.
7. Sometimes, the members are of such a shape (i.e. circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.
8. The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.
9. It is possible to weld any part of a structure at any point. But riveting requires enough clearance.
10. The process of welding takes less time than the riveting.

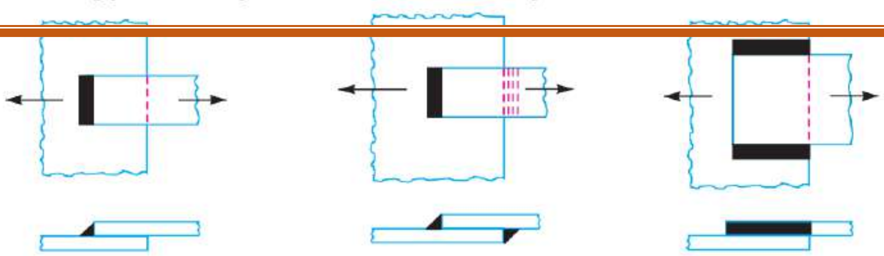
Types of Welded Joints:

Following two types of welded joints are important from the subject point of view:

1. Lap joint or fillet joint, and 2. Butt joint

1.Lap Joint

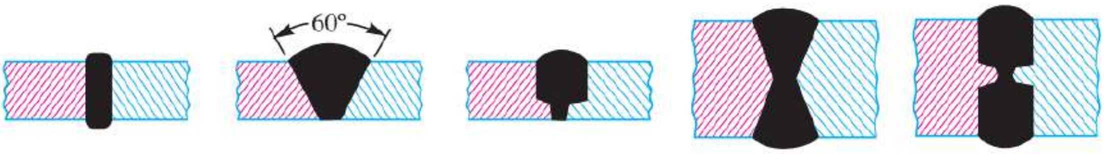
The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be
1. Single transverse fillet,
2. Double transverse fillet, and
3. Parallel fillet joints.



(a) Single transverse. (b) Double transverse. (c) Parallel fillet.

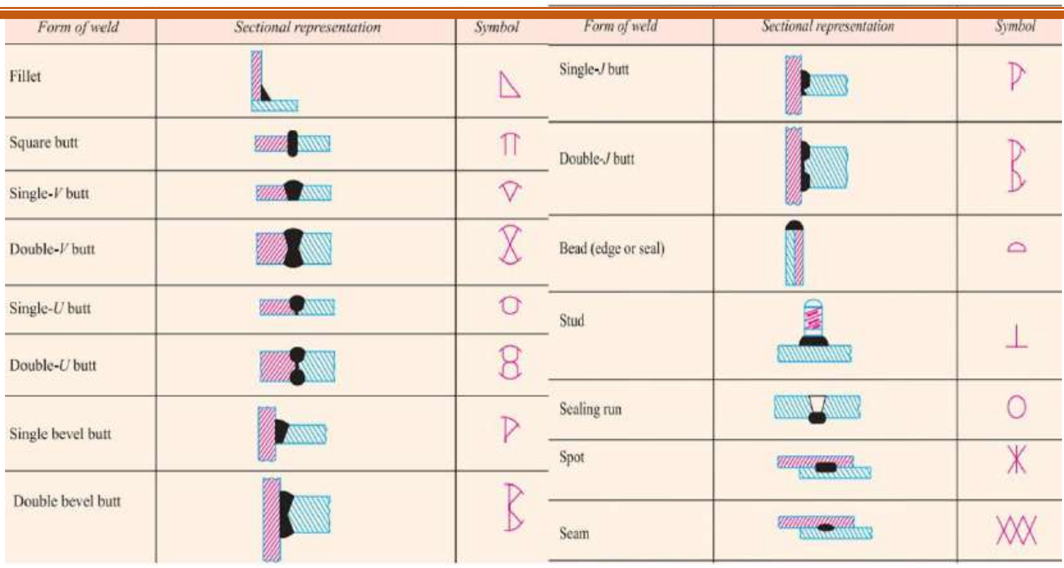
2.Butt joint

The butt joint is obtained by placing the plates edge to edge as shown in Fig. In butt welds, the plate edges do not require bevelling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U-groove on both sides.

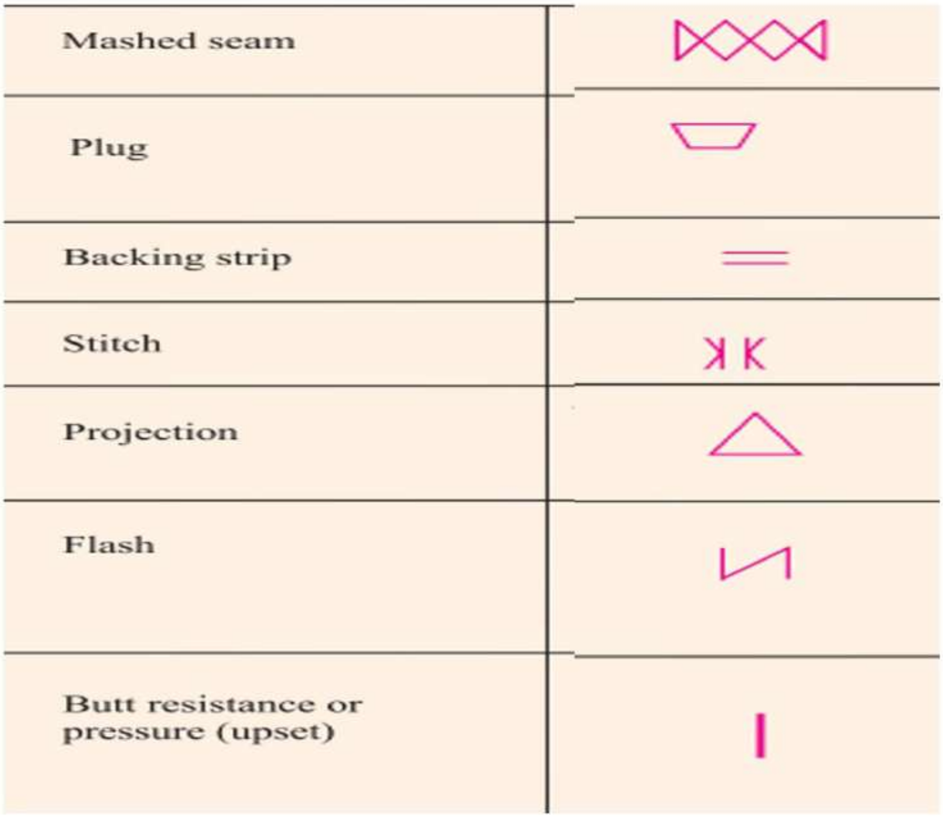


(a) Square butt joint. (b) Single V-butt joint. (c) Single U-butt joint. (d) Double V-butt joint. (e) Double U-butt joint.

Basic Weld Symbols:

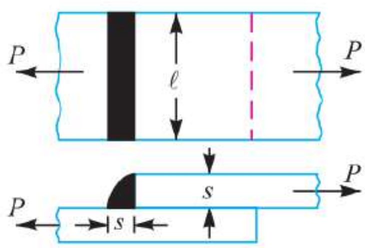


Form of weld	Sectional representation	Symbol
Fillet		
Square butt		
Single-*V* butt		
Double-*V* butt		
Single-*U* butt		
Double-*U* butt		
Single bevel butt		
Double bevel butt		
Single-*J* butt		
Double-*J* butt		
Bead (edge or seal)		
Stud		
Sealing run		
Spot		
Seam		

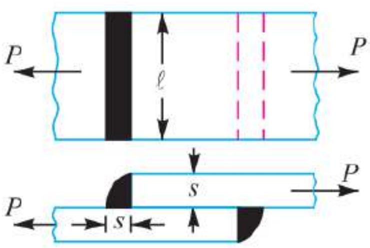


Mashed seam	
Plug	
Backing strip	
Stitch	
Projection	
Flash	
Butt resistance or pressure (upset)	

Strength of Transverse Fillet Welded Joints

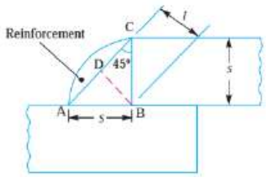


(*a*) Single transverse fillet weld.



(*b*) Double transverse fillet weld.

In order to determine the strength of the fillet joint, it is assumed that the section of fillet is aright angled triangle ABC with hypotenuse AC making equal angles with other two sides AB and BC. The enlarged view of the fillet is shown in Fig.. The length of each side is known as leg or size of the weld and the perpendicular distance of the hypotenuse from the intersection of legs (i.e. BD) is known as throat thickness. The minimum area of the weld is obtained at the throat BD, which is givenby the product of the throat thickness and length of weld.



Let t = Throat thickness (BD),
s = Leg or size of weld = Thickness of plate, and
l = Length of weld,
From Fig., we find that the throat thickness,
t = s × sin 45° = 0.707 s
∴ Minimum area of the weld or throat area,
A = Throat thickness × Length of weld = t × l = 0.707 s × l
If σ_t is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

P = Throat area × Allowable tensile stress = 0.707 s × l × σ_t

$$P = 0.707\ s \times l \times \sigma_t$$

and
tensile strength of the joint for double fillet weld,

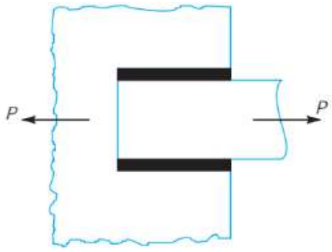
$$P = 2 \times 0.707\ s \times l \times \sigma_t$$

$$P = 1.414\ s \times l \times \sigma_t$$

Strength of Parallel Fillet Welded Joints:

The parallel fillet welded joints are designed for shear strength. Consider a double parallel fillet welded joint as shown in this figure, the minimum area of weld or the throat area

A= 0.707 S l



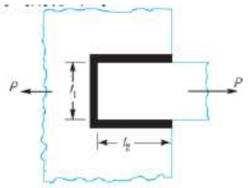
If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

P = Throat area × Allowable shear stress

$$P = 0.707\ s \times l \times \tau$$

shear strength of the joint for double parallel fillet weld

$$P = 1.414\ s \times l \times \tau$$



For combination of single transverse and double parallel fillet welds as shown in above figure:

$$P = 0.707s \times l_1 \times \sigma_t + 1.414\, s \times l_2 \times \tau$$

Eccentrically Loaded Welded Joints

When the shear and bending stresses are simultaneously present in a joint then maximum stresses are as follows

Maximum normal stress:

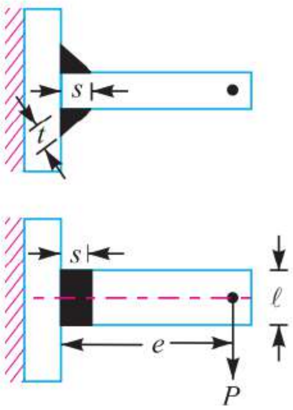
$$\sigma_{t(max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

maximum shear stress:

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

σ_b = Bending stress, and

τ = Shear stress.



T-joint fixed at one end and subjected to an eccentric load P at a distance e as shown have

s = Size of weld,

l = Length of weld, and

t = Throat thickness

The joint will be subjected to the following two types of stresses:

1. Direct shear stress due to the shear force P acting at the welds, and

2. Bending stress due to the bending moment P × e.

We know that area at the throat,

A = Throat thickness × Length of weld

A= t × l × = 2 × 0.707 s × l = 1.414 s × l

Now, Shear stress in the weld $\tau = \frac{P}{A} = \frac{P}{1.414\, s \times l}$

Bending stress:

$$, \sigma_b = \frac{M}{Z} = \frac{P \times e \times 4.242}{s \times l^2} = \frac{4.242\ P \times e}{s \times l^2}$$

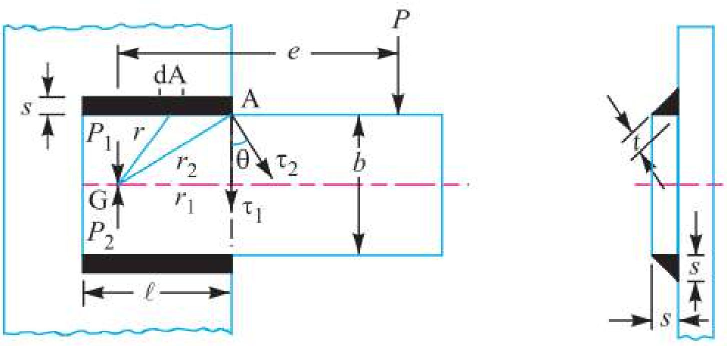
Where $Z = \frac{t \times l^2}{6} \times 2 \quad = \frac{0.707\, s \times l^2}{6} \times 2 = \frac{s \times l^2}{4.242}$

When a welded joint is loaded eccentrically as shown in Fig

the following two types of the stresses are induced:

1. Direct or primary shear stress, and

2. Shear stress due to turning moment



Let P = Eccentric load,

e = Eccentricity *i.e.* perpendicular distance between the line of action of load and centre of gravity (G) of the throat section or fillets,

l = Length of single weld,

s = Size or leg of weld, and

t = Throat thickness.

Let two loads P

1 and P2 (each equal to P) are introduced at the centre of gravity 'G' of the weld

system. The effect of load P1 = P is to produce direct shear stress which is assumed to be uniform over the entire weld length. The effect of load P2 = P is to produce a turning moment of magnitude P × e which tends of rotate the joint about the centre of gravity 'G' of the weld system. Due to the turning moment, secondary shear stress is induced.

We know that the direct or primary shear stress,

$$\tau_1 = \frac{\text{Load}}{\text{Throat area}} = \frac{P}{A} = \frac{P}{2\, t \times l}$$

$$= \frac{P}{2 \times 0.707\ s \times l} = \frac{P}{1.414\ s \times l}$$

... ($\because$ Throat area for single fillet weld = $t \times l = 0.707\ s \times$ 1)

Since the shear stress produced due to the turning moment ($T = P \times e$) at any section is proportional to its radial distance from G, therefore stress due to $P \times e$ at the point A is proportional to AG (r2) and is in a direction at right angles to AG. In other words,

$$\frac{\tau_2}{r_2} = \frac{\tau}{r} = \text{Constant}$$

or

$$\tau = \frac{\tau_2}{r_2} \times r \qquad \text{...}(i)$$

where τ_2 is the shear stress at the maximum distance (r_2) and τ is the shear stress at any distance r.

Consider a small section of the weld having area dA at a distance r from G.

∴ Shear force on this small section

$$= \tau \times dA$$

and turning moment of this shear force about G,

$$dT = \tau \times dA \times r = \frac{\tau_2}{r_2} \times dA \times r^2 \qquad \text{... [From equation } (i)]$$

∴ Total turning moment over the whole weld area,

$$T = P \times e = \int \frac{\tau_2}{r_2} \times dA \times r^2 = \frac{\tau_2}{r_2} \int dA \times r^2$$

$$= \frac{\tau_2}{r_2} \times J \qquad \left(\because J = \int dA \times r^2\right)$$

where J = Polar moment of inertia of the throat area about G.

∴ Shear stress due to the turning moment *i.e.* secondary shear stress,

$$\tau_2 = \frac{T \times r_2}{J} = \frac{P \times e \times r_2}{J}$$

In order to find the resultant stress, the primary and secondary shear stresses are combined vectorially.

∴ Resultant shear stress at A,

$$\tau_A = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \times \tau_2 \times \cos\theta}$$

where θ = Angle between τ_1 and τ_2, and

$$\cos\theta = r_1 / r_2$$

Note: The polar moment of inertia of the throat area (A) about the centre of gravity (G) is obtained by the parallel axis theorem, *i.e.*

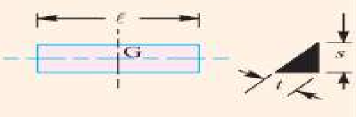
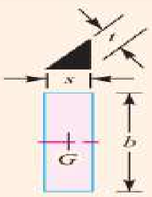
$$J = 2\,[I_{xx} + A \times x^2] \qquad \text{... } (\because \text{ of double fillet weld})$$

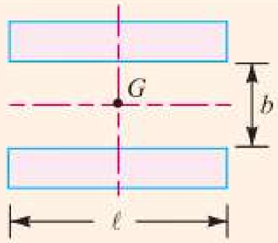
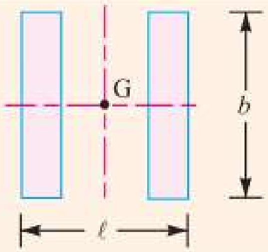
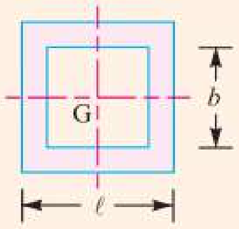
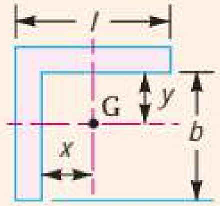
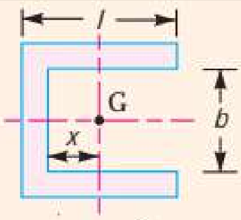
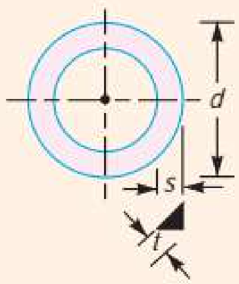
$$= 2\left[\frac{A \times l^2}{12} + A \times x^2\right] = 2A\left(\frac{l_2}{12} + x^2\right)$$

where A = Throat area = $t \times l = 0.707\, s \times l$,

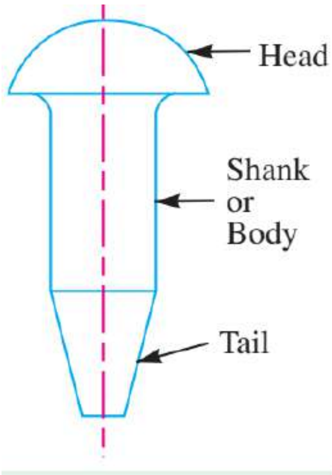
l = Length of weld, and

x = Perpendicular distance between the two parallel axes.

S.No	Type of weld	Polar moment of inertia (J)	Section modulus (Z)
1.		$\frac{t.l^3}{12}$	—
2.		$\frac{t.b^3}{12}$	$\frac{t.b^2}{6}$

S.No	Type of weld	Polar moment of inertia (J)	Section modulus (Z)
3.		$\frac{t.l(3b^2+l^2)}{6}$	$t.b.l$
4.		$\frac{t.b(b^2+3l^2)}{6}$	$\frac{t.b^2}{3}$
5.		$\frac{t(b+l)^3}{6}$	$t\left(b.l+\frac{b^2}{3}\right)$
6.	 $x=\frac{l^2}{2(l+b)}, y=\frac{b^2}{2(l+b)}$	$t\left[\frac{(b+l)^4-6b^2l^2}{12(l+b)}\right]$	$t\left(\frac{4l.b+b^2}{6}\right)$ (Top) $t\left[\frac{b^2(4lb+b)}{6(2l+b)}\right]$ (Bottom)
7.	 $x=\frac{l^2}{2l+b}$	$t\left[\frac{(b+2l)^3}{12}-\frac{l^2(b+l)^2}{b+2l}\right]$	$t\left(l.b+\frac{b^2}{6}\right)$
8.		$\frac{\pi t d^3}{4}$	$\frac{\pi t d^2}{4}$

Riveted Joints



A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called shank or body and lower portion of shank is known as tail.

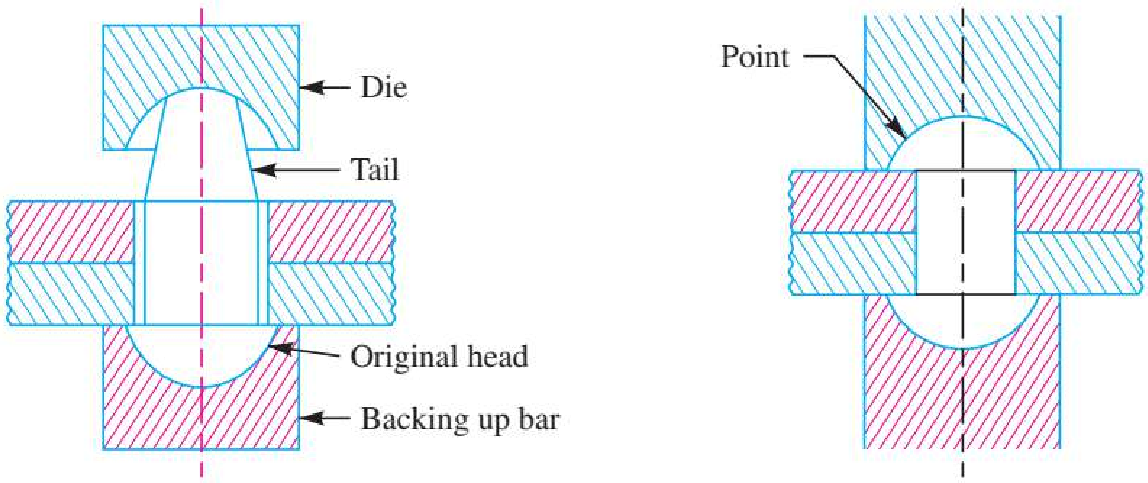
The rivets are used to make permanent fastening between the plates such as in structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metals.

The fastenings (i.e. joints) may be classified into the following two groups.

The permanent fastenings are those fastenings which can not be disassembled without destroying the connecting components. The examples of permanent fastenings in order of strengthare soldered, brazed, welded and riveted joints. The temporary or detachable fastenings are those fastenings which can be disassembled without destroying the connecting components. The examples of temporary fastenings are screwed, keys, cotters, pins and splined joints.

Methods of Riveting

The function of rivets in a joint is to make a connection that has strength and tightness. The strength is necessary to prevent failure of the joint. The tightness is necessary in order to contribute to strength and to prevent leakage as in a boiler or in a ship hull. When two plates are to be fastened together by a rivet as shown in Fig. the holes in the plates are punched and reamed or drilled. Punching is the cheapest method and is used for relatively thin plates and in structural work. Since punching injures the material around the hole, therefore drilling is used in most pressure-vessel work. In structural and pressure vessel riveting, the diameter of the rivet hole is usually 1.5 mm larger than the nominal diameter of the rivet.



(*a*) Initial position. (*b*) Final position.

The plates are drilled together and then separated to remove any burrs or chips so as to have a tight flush joint between the plates. A cold rivet or a red hot rivet is introduced into the plates and the point (i.e. second head) is then formed. When a cold rivet is used, the process is known

as cold riveting and when a hot rivet is used, the process is known as hot riveting. The cold riveting process s used for structural joints while hot riveting is used to make leak proof joints.

The riveting may be done by hand or by a riveting machine. In hand riveting, the original rivet head is backed up by a hammer or heavy bar and then the die or set, as shown in Fig. 9.2 (a), is placed against the end to be headed and the blows are applied by a hammer. This causes the shank to expand thus filling the hole and the tail is converted into a point as shown in Fig. 9.2 (b). As the rivet cools, it tends to contract. The lateral contraction will be slight, but there will be a longitudinal tension introduced in the rivet which holds the plates firmly together

In machine riveting, the die is a part of the hammer which is operated by air, hydraulic or steam pressure.

The material of the rivets must be tough and ductile. They are usually made of steel (low carbon steel or nickel steel), brass, aluminium or copper, but when strength and a fluid tight joint is the main consideration, then the steel rivets are used.

Types of Riveted Joints

1. Lap Joint

A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.

2. Butt Joint

A butt joint is that in which the main plates are kept in alignment butting (i.e. touching) each other and a cover plate (i.e. strap) is placed either on one side or on both sides of the main plates. The cover plate is then riveted together with the main plates. Butt joints are of the following two types :

1. Single strap butt joint, and 2. Double strap butt joint.

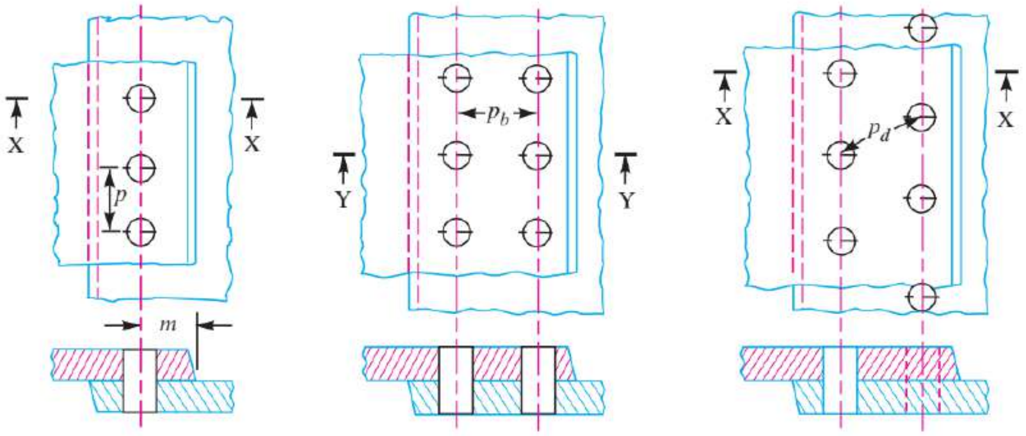
In a single strap butt joint, the edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

In a double strap butt joint, the edges of the main plates butt against each other and two cover plates are placed on both sides of the main plates and then riveted together. In addition to the above, following are the types of riveted joints depending upon the number of rows of the rivets.

1. Single riveted joint, and 2. Double riveted joint.

A single riveted joint is that in which there is a single row of rivets in a lap joint as shown in Fig. and there is a single row of rivets on each side in a butt joint as shown in Fig..

A double riveted joint is that in which there are two rows of rivets in a lap joint as shown in Fig. there are two rows of rivets on each side in a butt joint as shown in Fig.



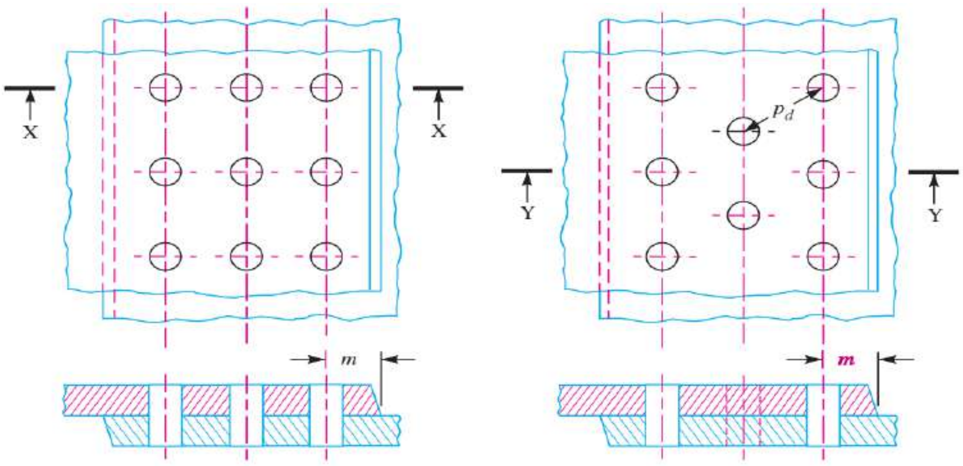
(*a*) Single riveted lap joint.

(*b*) Double riveted lap joint (Chain riveting).

(*c*) Double riveted lap joint (Zig-zag riveting).

When the rivets in the various rows are opposite to each other, as shown in Fig. then the joint is said to be chain riveted. On the other hand, if the rivets in the adjacent rows are staggered in such a way that

every rivet is in the middle of the two rivets of the opposite row as shown in Fig. 9.6 (c), then the joint is said to be zig-zag riveted.,



(*a*) Chain riveting.

(*b*) Zig-zag riveting.

Fig. 9.7. Triple riveted lap joint.

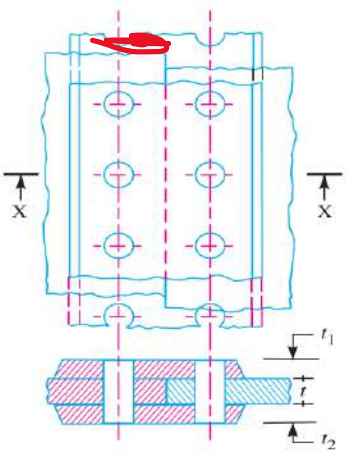
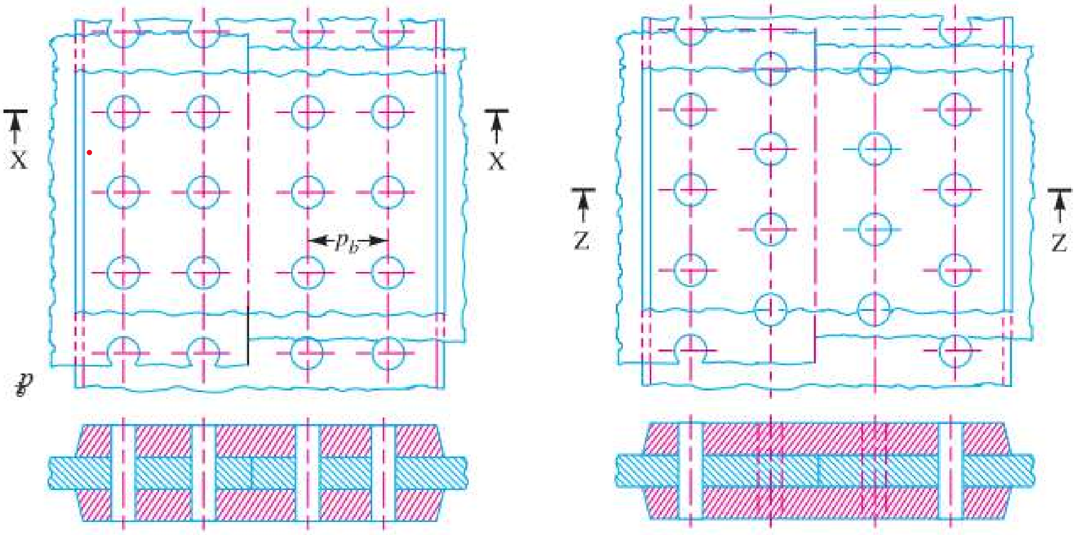


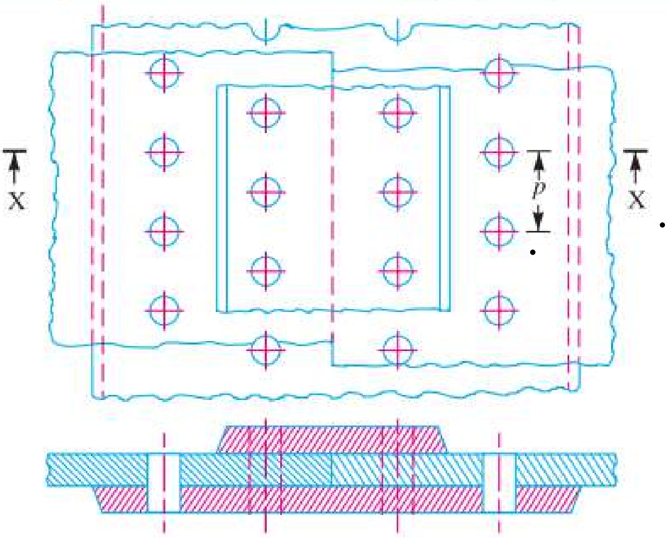
Fig. 9.8. Single riveted double strap butt joint.



(a) Chain riveting.

(b) Zig-zag riveting.

[redacted]. Double riveted double strap (equal) butt joints.



Important Terms Used in Riveted Joints

1. Pitch. It is the distance from the centre of one rivet to the centre of the next rivet measured parallel to the seam as shown in Fig. It is usually denoted by p.

2. Back pitch. It is the perpendicular distance between the centre lines of the successive rows as shown in Fig. It is usually denoted by P_b.

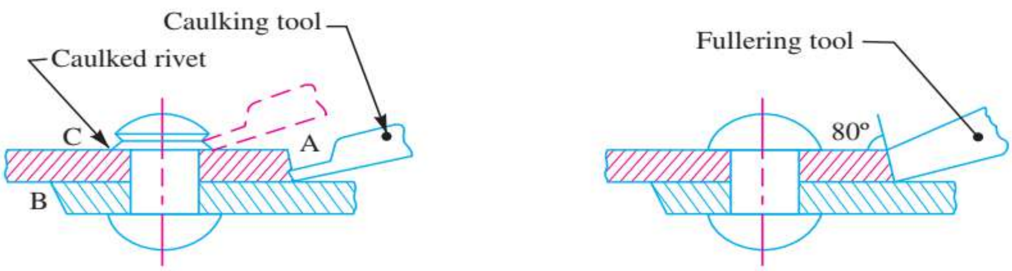
3. Diagonal pitch. It is the distance between the centres of the rivets in adjacent rows of zig-zag riveted joint as shown in Fig. It is usually denoted by P_d.

4. Margin or marginal pitch. It is the distance between the centre of rivet hole to the nearest edge of the plate as shown in Fig. It is usually denoted by m.

Caulking and Fullering

In order to make the joints leak proof or fluid tight in pressure vessels like steam boilers, air receivers and tanks etc. a process known as caulking is employed. In this process, a narrow blunt tool called caulking tool, about 5 mm thick and 38 mm in breadth, is used. The edge of the tool is ground to an angle of 80°. The tool is moved after each blow along the edge of the plate, which is planed to a bevel of 75° to 80° to facilitate the forcing down of edge. It is seen that the tool burrs down the plate at A in Fig. (a) forming a metal to metal joint. In actual practice, both the edges at A and B are caulked. The head of the rivets as shown at C are also turned down with a caulking tool to make a joint steam tight. A great care is taken to prevent injury to the plate below the tool.

A more satisfactory way of making the joints staunch is known as fullering which has largely superseded caulking. In this case, a fullering tool with a thickness at the end equal to that of the plate is used in such a way that the greatest pressure due to the blows occur near the joint, giving a clean finish, with less risk of damaging the plate. A fullering process is shown in Fig. (b).

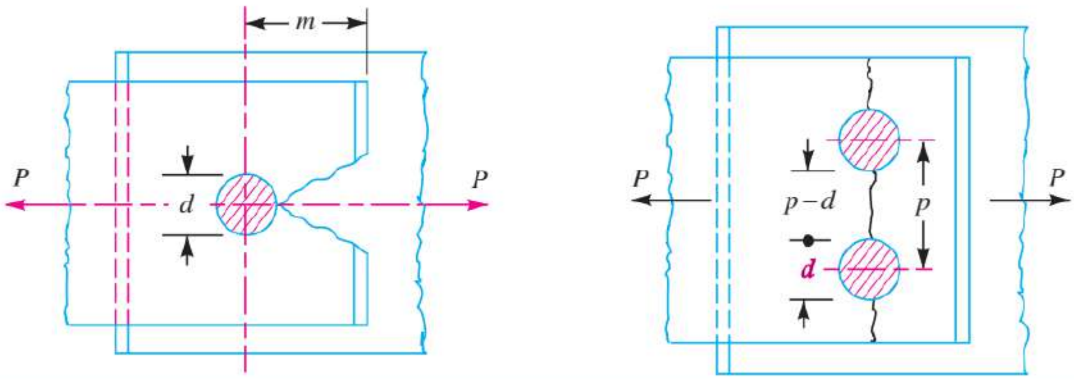


(a) Caulking.   (b) Fullering.

Failures of a Riveted Joint

A riveted joint may fail in the following ways :

1. **Tearing of the plate at an edge**. A joint may fail due to tearing of the plate at an edge as shown in Fig.. This can be avoided by keeping the margin, **m = 1.5d**, where d is the diameter of the rivet hole.



[redacted]. Tearing of the plate at an edge.   [redacted]. Tearing of the plate across the rows of rivets.

2. **Tearing of the plate across a row of rivets**. Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig above In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only. The resistance offered by the plate against tearing is known as tearing resistance or tearing strength or tearing value of the plate.

Let p = Pitch of the rivets,

d = Diameter of the rivet hole,

t = Thickness of the plate, and

σ_t = Permissible tensile stress for the plate material.

We know that tearing area per pitch length,

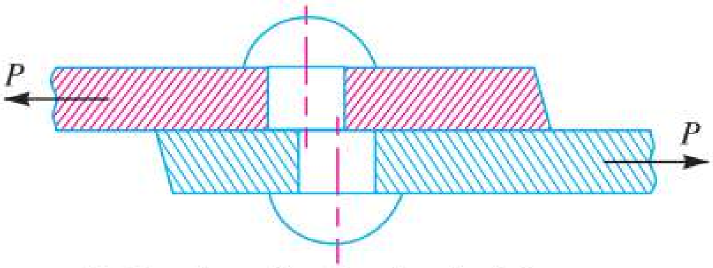
$$A_t = (p - d)t$$

∴ Tearing resistance or pull required to tear off the plate per pitch length,

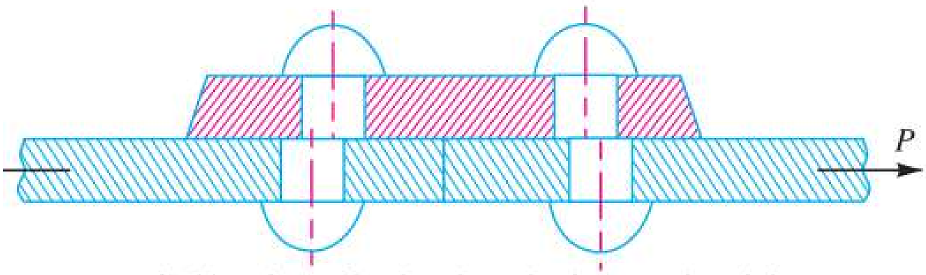
$$P_t = A_t.\sigma_t = (p - d)t.\sigma_t$$

When the tearing resistance (P_t) is greater than the applied load (P) per pitch length, then this type of failure will not occur.

3. **Shearing of the rivets**. The plates which are connected by the rivets exert tensile stress on the rivets, and if the rivets are unable to resist the stress, they are sheared off as shown in Fig. It may be noted that the rivets are in *single shear in a lap joint and in a single cover butt joint, as shown in Fig. But the rivets are in double shear in a double cover butt joint as shown in Fig. The resistance offered by a rivet to be sheared off is known as shearing resistance or shearing strength or shearing value of the rivet.



(a) Shearing off a rivet in a lap joint.



(b) Shearing off a rivet in a single cover butt joint.

[redacted]. Shearing of rivets.

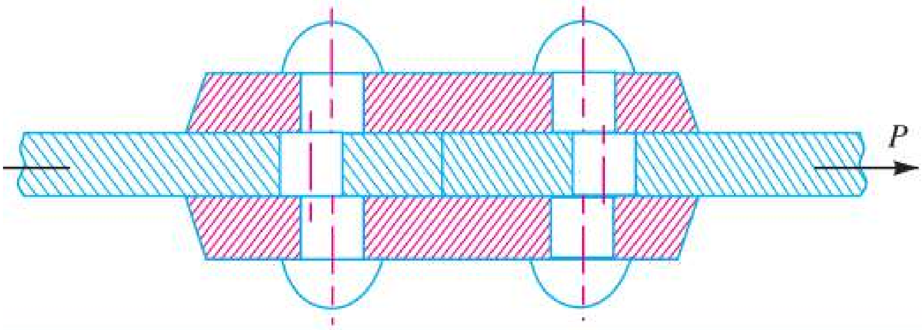


Fig. [redacted]. Shearing off a rivet in double cover butt joint.

Let d = Diameter of the rivet hole,

τ = Safe permissible shear stress for the rivet material, and

n = Number of rivets per pitch length.

n = Number of rivets per pitch length.

We know that shearing area,

$$A_s = \frac{\pi}{4} \times d^2 \qquad \text{...(In single shear)}$$

$$= 2 \times \frac{\pi}{4} \times d^2 \qquad \text{...(Theoretically, in double shear)}$$

$$= 1.875 \times \frac{\pi}{4} \times d^2 \qquad \text{...(In double shear, according to Indian Boiler Regulations)}$$

∴ Shearing resistance or pull required to shear off the rivet per pitch length,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau \qquad \text{...(In single shear)}$$

$$= n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau \qquad \text{...(Theoretically, in double shear)}$$

$$= n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau \qquad \text{...(In double shear, according to Indian Boiler Regulations)}$$

When the shearing resistance (P_s) is greater than the applied load (P) per pitch length, then this type of failure will occur.

4. Crushing of the plate or rivets. Sometimes, the rivets do not actually shear off under the tensile stress, but are crushed as shown in Fig. Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose. The failure of rivets in such a manner is also known as bearing failure. The area which resists this action is the projected area of the hole or rivet on diametral plane

The resistance offered by a rivet to be crushed is known as ***crushing resistance*** or ***crushing strength*** or ***bearing value*** of the rivet.

Let d = Diameter of the rivet hole,

t = Thickness of the plate,

σ_c = Safe permissible crushing stress for the rivet or plate material, and

n = Number of rivets per pitch length under crushing.

We know that crushing area per rivet (*i.e.* projected area per rivet),

$$A_c = d.t$$

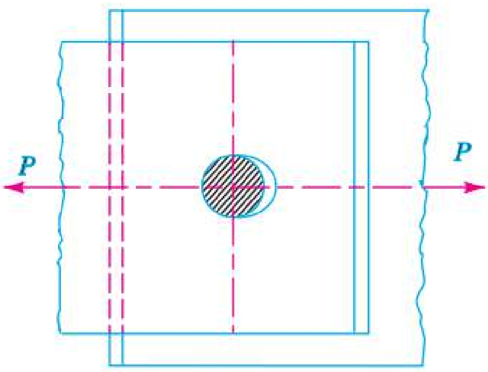
∴ Total crushing area $= n.d.t$

and crushing resistance or pull required to crush the rivet per pitch length,

$$P_c = n.d.t.\sigma_c$$

When the crushing resistance (P_c) is greater than the applied load (P) per pitch length, then this type of failure will occur.

The number of rivets under shear shall be equal to the number of rivets under crushing.



Crushing of a rivet.

Efficiency of a Riveted Joint

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate.

strength of the riveted joint= Least of P_t , P_s and P_c .

Strength of the un-riveted or solid plate per pitch length,

$$P = p \times t \times \sigma_t$$

∴ Efficiency of the riveted joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{p \times t \times \sigma_t}$$

where p = Pitch of the rivets,

t = Thickness of the plate, and

σ_t = Permissible tensile stress of the plate material.

Keys

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

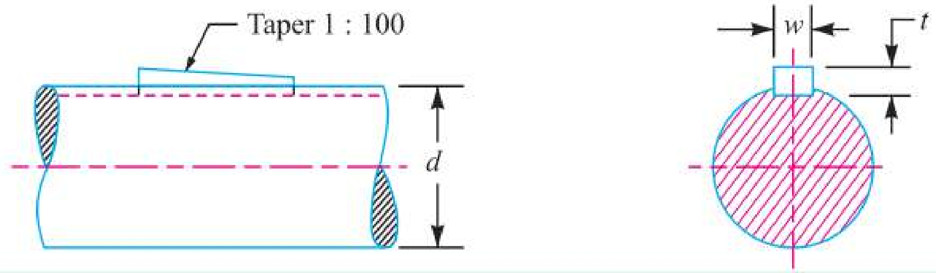
1. Sunk keys, 2. Saddle keys, 3. Tangent keys, 4. Round keys, and 5. Splines.

Rectangular sunk key. A rectangular sunk key is shown in Fig. The usual proportions of this key are,

Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$

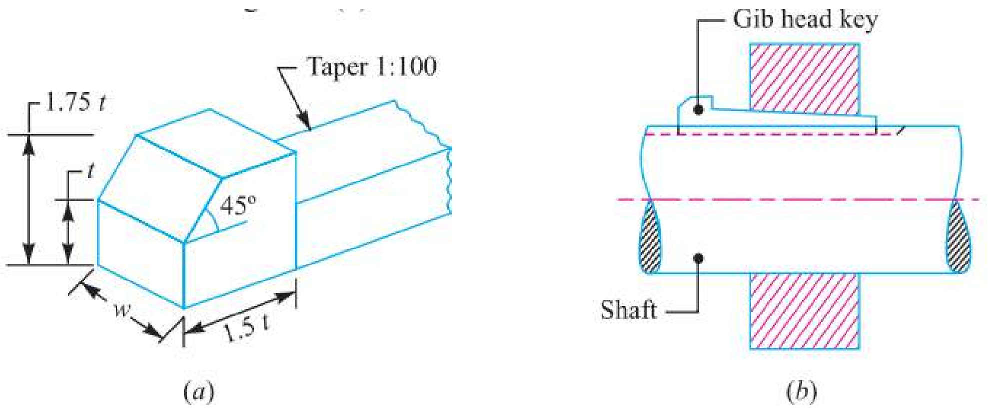
where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.



Square sunk key. The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e. w = t = d / 4.

Gib-head key. It is a rectangular sunk key with a head at one end known as gib head. It is usually provided to facilitate the removal of key. A gib head key is shown in Fig,



The usual proportions of the gib head key are :

Width, w = d / 4 ;

and thickness at large end, t = 2w / 3 = d / 6

Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. Forces (F1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key. The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub. The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig. In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

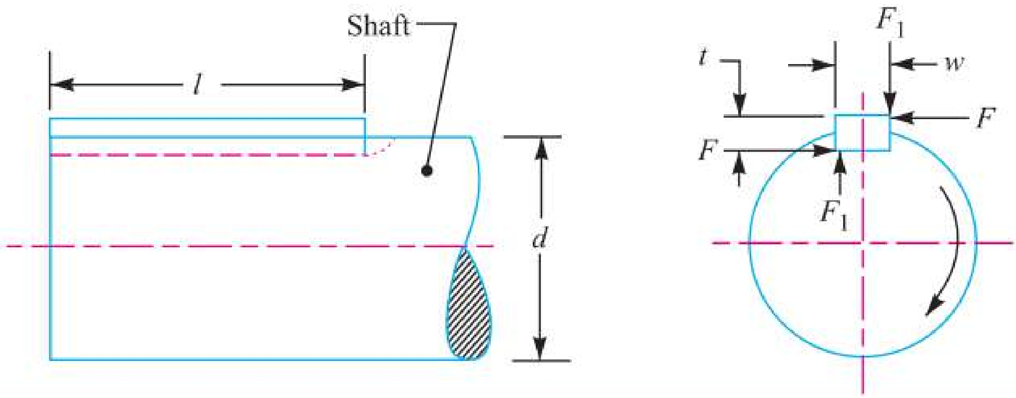


Fig. [redacted]. Forces acting on a sunk key.

Strength of a Sunk Key

A key connecting the shaft and hub is shown in above Figure:

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$$

$\therefore$ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \qquad \text{...}(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

$\therefore$ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{...}(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{...[Equating equations } (i) \text{ and } (ii)]$$

or

$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \qquad \text{...}(iii)$$

The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress. Therefore from equation *(iii)*, we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{...}(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \qquad \text{...}(v)$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations *(iv)* and *(v)*, we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \qquad \text{... (Taking } w = d/4) \quad \text{...}(vi)$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$$\therefore \quad l = 1.571\, d \qquad \text{... [From equation } (vi)]$$

Shaft strength factor

It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway, denoted by '**e**'**.**

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

h = Depth of keyway = $\frac{\text{Thickness of key } (t)}{2}$

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation :

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

where k_θ = Reduction factor for angular twist.

Chapter 3 : Design of Shaft and Key

Design of Shaft

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft.

1. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.
2. An axle, though similar in shape to the shaft, is a stationary machine element and is used for thetransmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum,a car wheel or a rope sheave.
3. A spindle is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. lathe spindles).

Material Used for Shafts

The material used for shafts should have the following properties :

1. It should have high strength.

2. It should have good machinability.

3. It should have low notch sensitivity factor.

4. It should have good heat treatment properties.

5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as

(a) 112 MPa for shafts without allowance for keyways.

(b) 84 MPa for shafts with allowance for keyways.

The maximum permissible shear stress may be taken as

(a) 56 MPa for shafts without allowance for key ways.

(b) 42 MPa for shafts with allowance for keyways.

Design of Shafts

The shafts may be designed on the basis of

1. Strength, and 2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

a. Shafts Subjected to Twisting Moment Only

We know that

$$\frac{T}{J} = \frac{\tau}{r} \quad \ldots(i)$$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre

= $d / 2$; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation *(i)* may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \quad \ldots(ii)$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation *(i)*, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \quad \ldots(iii)$$

Let k = Ratio of inside diameter and outside diameter of the shaft

$= d_i / d_o$

Now the equation *(iii)* may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \quad \ldots(iv)$$

From the equations *(iii)* or *(iv)*, the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2) R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

b. Shafts Subjected to Bending Moment Only

We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{...}(i)$$

where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation *(i)*, we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64} \left[(d_o)^4 - (d_i)^4 \right] = \frac{\pi}{64} (d_o)^4 (1 - k^4) \quad \text{...(where } k = d_i / d_o\text{)}$$

and $y = d_o / 2$

Again substituting these values in equation *(i)*, we have

$$\frac{M}{\frac{\pi}{64} (d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

c. Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine’s theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σ_b from Art. 14.9 and Art. 14.10, we have

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad ...(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation ***(i)*** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad ...(iii)$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or

$$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad ...(iv)$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress $[\sigma_{b(max)}]$ equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad ...(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. ***Torsional rigidity***. The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G \cdot \theta}{L} \quad \text{or} \quad \theta = \frac{T \cdot L}{J \cdot G}$$

where
θ = Torsional deflection or angle of twist in radians,
T = Twisting moment or torque on the shaft,
J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \qquad ...(\text{For solid shaft})$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \qquad ...(\text{For hollow shaft})$$

G = Modulus of rigidity for the shaft material, and
L = Length of the shaft.

2. ***Lateral rigidity***. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then,

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Chapter 4: Design of Coupling

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following.

1.To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.

2. To provide for misalignment of the shafts or to introduce mechanical flexibility.

3. To reduce the transmission of shock loads from one shaft to another.

4. To introduce protection against overloads.

5. It should have no projecting parts.

Requirements of a Good Shaft Coupling

A good shaft coupling should have the following requirements :

1. It should be easy to connect or disconnect.

2. It should transmit the full power from one shaft to the other shaft without losses.

3. It should hold the shafts in perfect alignment.

4. It should reduce the transmission of shock loads from one shaft to another shaft.

5. It should have no projecting parts.

Types of Shafts Couplings

Shaft couplings are divided into two main groups as follows :

1. ***Rigid coupling***. It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are

(a) Sleeve or muff coupling.

(b) Clamp or split-muff or compression coupling, and

(c) Flange coupling.

2. ***Flexible coupling***. It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are

(a) Bushed pin type coupling,

(b) Universal coupling, and

(c) Oldham coupling.

Sleeve or Muff-coupling

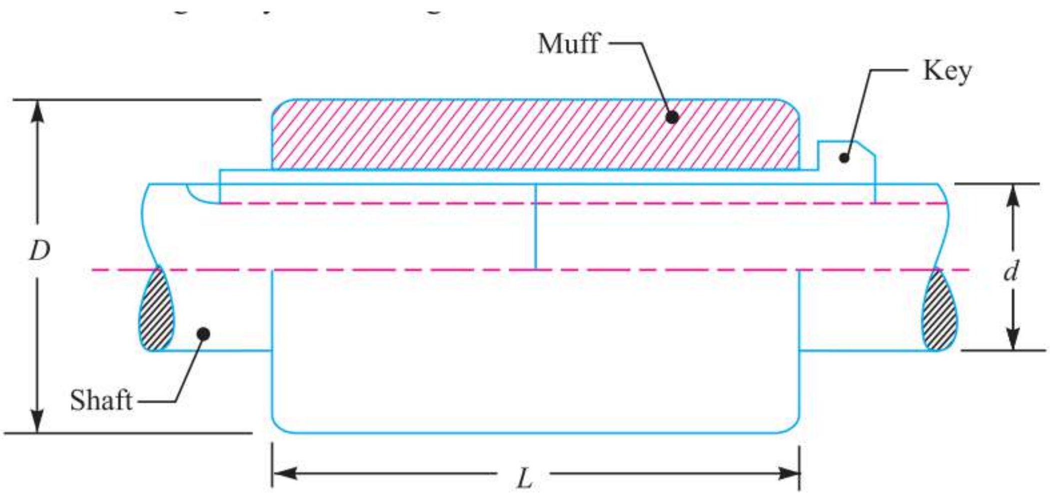
It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in the below figure. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque. The usual proportions of a cast iron sleeve coupling are as follows :

Outer diameter of the sleeve, $D = 2d + 13$ mm

and length of the sleeve, $L = 3.5\ d$

where d is the diameter of the shaft.

In designing a sleeve or muff-coupling, the following procedure may be adopted.



Design for sleeve

The sleeve is designed by considering it as a hollow shaft.

Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast rion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 \, (1 - k^4) \qquad \ldots (\because \; k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

Design for key

The length of the coupling key is atleast equal to the length of the sleeve (i.e. 3.5 d). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\,d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \ldots \text{(Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \ldots \text{(Considering crushing of the key)}$$

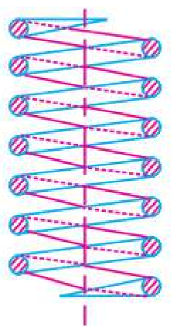
The Key is considered as gib head key and the proportions are:

Width, w = d / 4 ;

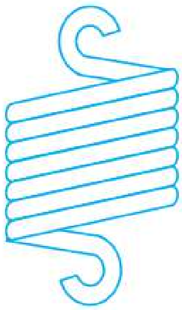
and thickness at large end, t = 2w / 3 = d / 6

Design a closed coil helical spring

The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are compression helical spring and tension helical spring as shown in Fig.



(a) Compression helical spring.



(b) Tension helical spring.

The helical springs are said to be closely coiled when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.

The helical springs have the following advantages:

(a) These are easy to manufacture.

(b) These are available in wide range.

(c) These are reliable.

(d) These have constant spring rate.

(e) Their performance can be predicted more accurately.

(f) Their characteristics can be varied by changing dimensions

Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant. It largely depends upon the service for which they are used i.e. severe service, average service or light service.

Severe service means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.

Average service includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.

Light service includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 per cent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.

The helical springs are either cold formed or hot formed depending upon the size of the wire. Wires of small sizes (less than 10 mm diameter) are usually wound cold whereas larger size wires are wound hot. The strength of the wires varies with size, smaller size wires have greater strength and less ductility, due to the greater degree of cold working.

Standard Size of Spring Wire

SWG	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*
7/0	12.70	7	4.470	20	0.914	33	0.2540
6/0	11.785	8	4.064	21	0.813	34	0.2337
5/0	10.973	9	3.658	22	0.711	35	0.2134
4/0	10.160	10	3.251	23	0.610	36	0.1930
3/0	9.490	11	2.946	24	0.559	37	0.1727
2/0	8.839	12	2.642	25	0.508	38	0.1524
0	8.229	13	2.337	26	0.457	39	0.1321
1	7.620	14	2.032	27	0.4166	40	0.1219
2	7.010	15	1.829	28	0.3759	41	0.1118
3	6.401	16	1.626	29	0.3454	42	0.1016
4	5.893	17	1.422	30	0.3150	43	0.0914
5	5.385	18	1.219	31	0.2946	44	0.0813
6	4.877	19	1.016	32	0.2743	45	0.0711

Terms used in Compression Springs

The following terms used in connection with compression springs are important from the subject point of view.

1. Solid length. When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be solid. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

   Solid length of the spring,

   $$L_S = n'.d$$

   where n' = Total number of coils, and

   d = Diameter of the wire.

2. Free length. The free length of a compression spring, as shown in Fig. 23.6, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed). Mathematically,

Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$= n'.d + \delta_{max} + 0.15\,\delta_{max}$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. Spring index. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically

Spring index, $C = D / d$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. Spring rate. The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

5. Pitch. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $p = \dfrac{\text{Free length}}{n' - 1}$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $p = \dfrac{L_F - L_S}{n'} + d$

where L_F = Free length of the spring,

L_S = Solid length of the spring,

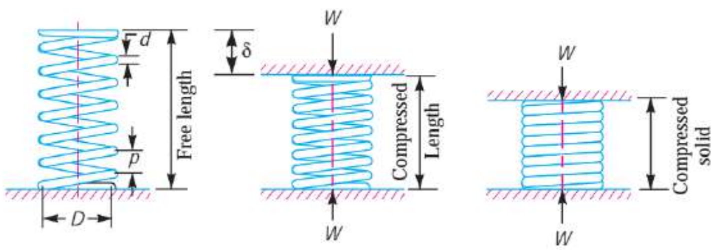
n' = Total number of coils, and

d = Diameter of the wire.

In choosing the pitch of the coils, the following points should be noted :

(*a*) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the yield point stress in torsion.

(*b*) The spring should not close up before the maximum service load is reached.



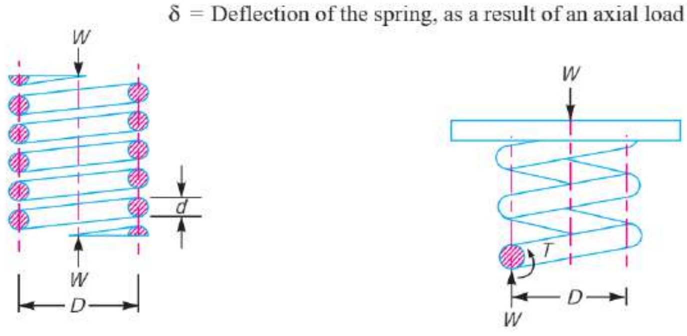
Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown

Let D = Mean diameter of the spring coil,
d = Diameter of the spring wire,
n = Number of active coils,
G = Modulus of rigidity for the spring material,
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,
C = Spring index = D/d,
p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W.



(*a*) Axially loaded helical spring.

(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Now consider a part of the compression spring as shown in Fig. (b). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire. A little consideration will show that part of the spring, as shown in Fig.(b), is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8 W.D}{\pi d^3} \qquad \ldots(i)$$

In addition to the torsional shear stress ($\tau 1$) induced in the wire, the following stresses also act on the wire :

1. Direct shear stress due to the load W, and
2. Stress due to curvature of wire.

We know that direct shear stress due to the load W,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2} \qquad \ldots(ii)$$

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8 W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The positive sign is used for the inner edge of the wire and negative sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8 W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8 W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\,d^3} \quad ...(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \frac{1}{2C}$

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \quad ...(iv)$$

where $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$

Deflection of Helical Springs of Circular Wire

Total active length of the wire,

l = Length of one coil × No. of active coils $= \pi\,D \times n$

Let θ = Angular deflection of the wire when acted upon by the torque T.

∴ Axial deflection of the spring,

$$\delta = \theta \times D/2 \quad ...(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \quad ...\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \frac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi\,D.n}{\frac{\pi}{32} \times d^4\,G} = \frac{16\,W.D^2.n}{G.d^4} \quad ...(ii)$$

Substituting this value of θ in equation (*i*), we have

$$\delta = \frac{16\,W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\,W.D^3.n}{G.d^4} = \frac{8\,W.C^3.n}{G.d} \quad ...(\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n} = \frac{G.d}{8\,C^3.n} = \text{constant}$$

Surge in spring.

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called ***surge.***

It has been found that the natural frequency of spring should be atleast twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n}\sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.